THREE HINGED PARABOLIC ARCHES

“An arch can be defined as a humped or curved beam subjected to transverse and other loads as well as the horizontal thrust at the supports.”

- It is simplest type of arch, consists of two section hinged at the crown and a hinge at support.
- The hinges at the support makes the ends of the arch to be fixed in position but not in direction.
- It is statically determinate structure.

Types of arches

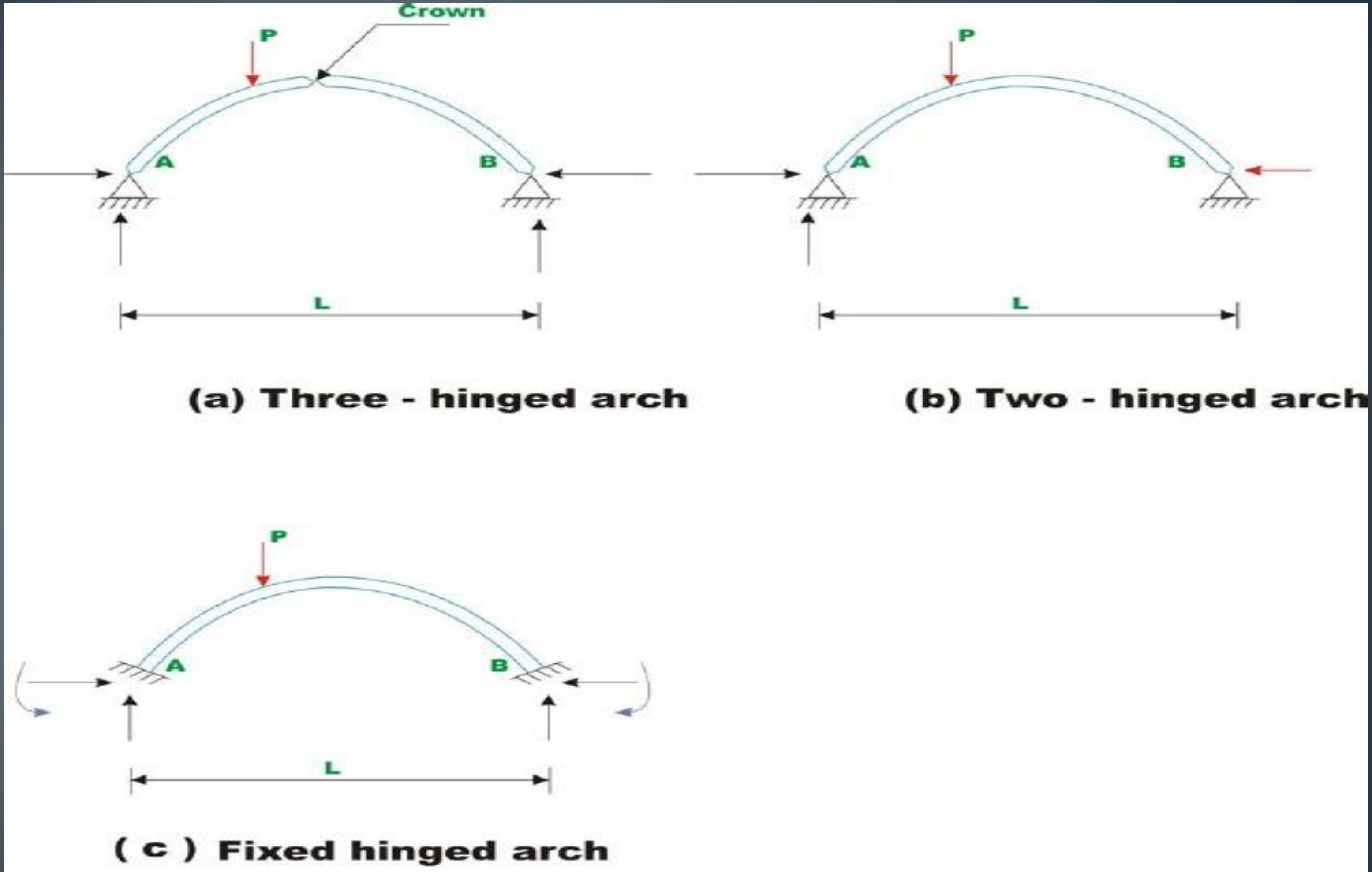
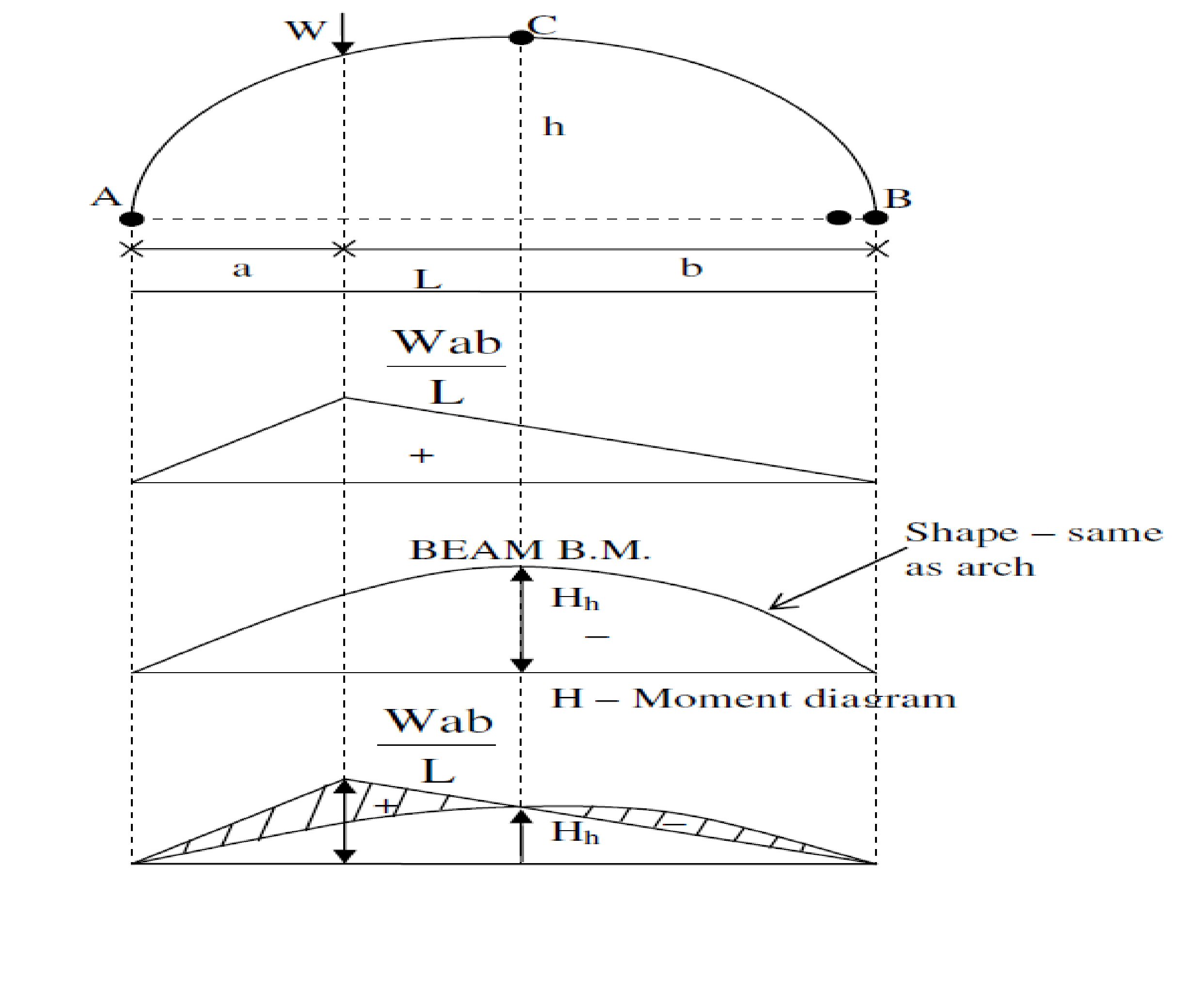


Fig. 32.4 Types of arches.

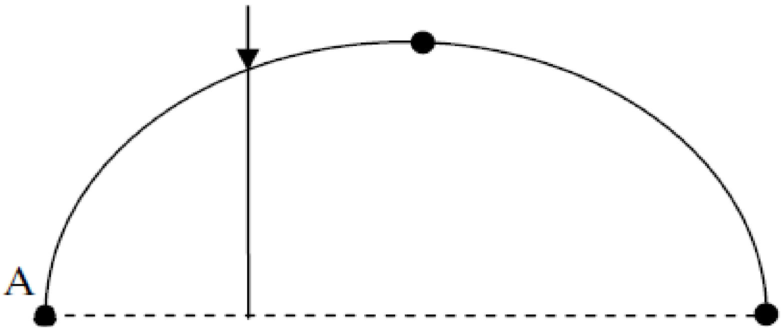
Arch v/s beam

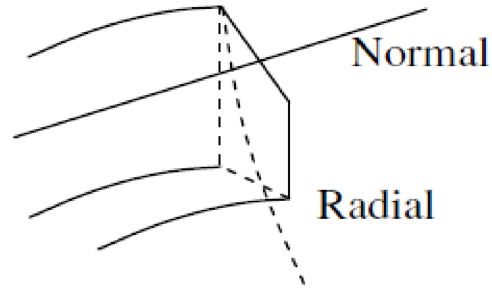
- In case of beams supporting UDL, the maximum bending moment increases with the square of the span and hence they become uneconomical for long span structures. In such situations arches could be advantageously employed, as they would develop horizontal reactions, which reduces the design bending moment.
- Arches are mainly used in bridge construction and doorways. In earlier days arches were constructed using stones and bricks. In modern times they are being constructed of reinforced concrete and steel.

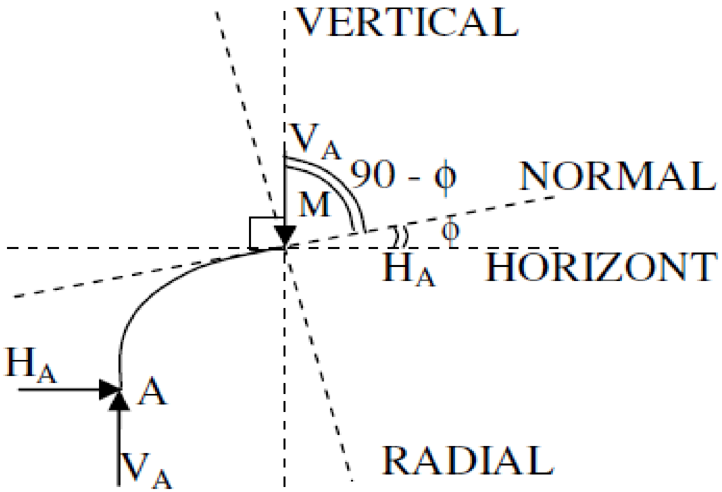
Bending moment diagram for 3–hinged arch: We know that for an arch, bending moment at any point is equal to beam BM-Hy(Refer comparison between arch and beam). Hy is called H-Moment. It varies with respect to Y. Therefore the shape of BM due to Hy should be the shape of the arch. Therefore to draw the BMD for an arch, draw the BMD for the beam over that superimpose the H-moment diagram as shown in fig.

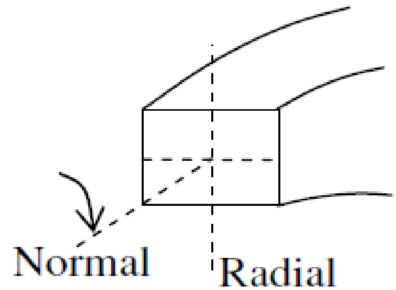


Normal thrust and radial shear in an arch









Total force acting along the normal is called normal thrust and total force acting along the radial direction is called radial shear. For the case shown in fig normal thrust

$= + H_A \cos \phi + V_A \cos (90 - \phi)$

$= H_A \cos \phi + V_A \sin \phi$

(Treat the force as +ve if it is acting towards the arch and -ve if it is away from the arch).

Radial shear $= + H_A \sin \phi - V_A \sin (90 - \phi)$

$= H_A \sin \phi + V_A \cos \phi$

(Treat force up the radial direction +ve and down the radial direction as -ve).

Equation of parabola is

$$y = \frac{4hx}{L^2}(L - x)$$

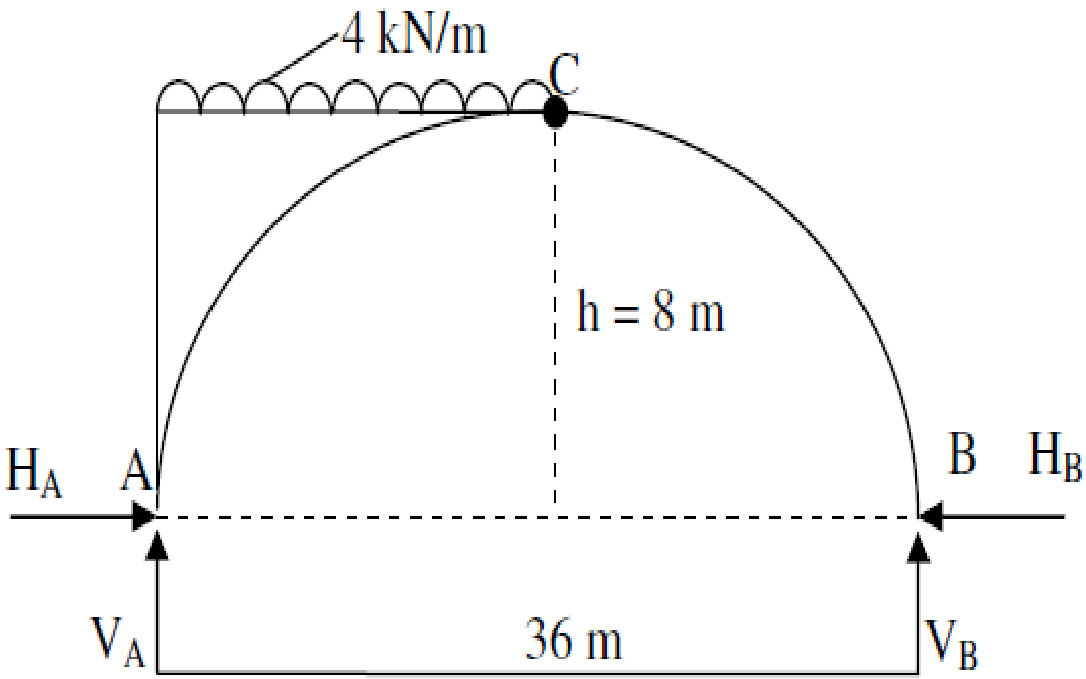
Rise equation

$$\tan\phi = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$

$$\tan\phi = \frac{4h}{L^2}(L - 2x)$$

Slope equation

1. A UDL of 4kN/m covers left half span of 3-hinged parabolic arch of span 36m and central rise 8m. Determine the horizontal thrust also find (i) BM (ii) Shear force (iii) Normal thrust (iv) Radial shear at the loaded quarter point. Sketch BMD.



$$\sum F_x = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B \qquad \text{------ (1)}$$

$$\sum F_y = 0$$

$$V_A + V_B = 4 \text{ x } 18$$

$$V_A + V_B = 72 \qquad \text{------ (2)}$$

$$\sum M_A = 0$$

$$-V_B \text{ x } 36 + 4 \text{ x } 18 \text{ x } 9 = 0$$

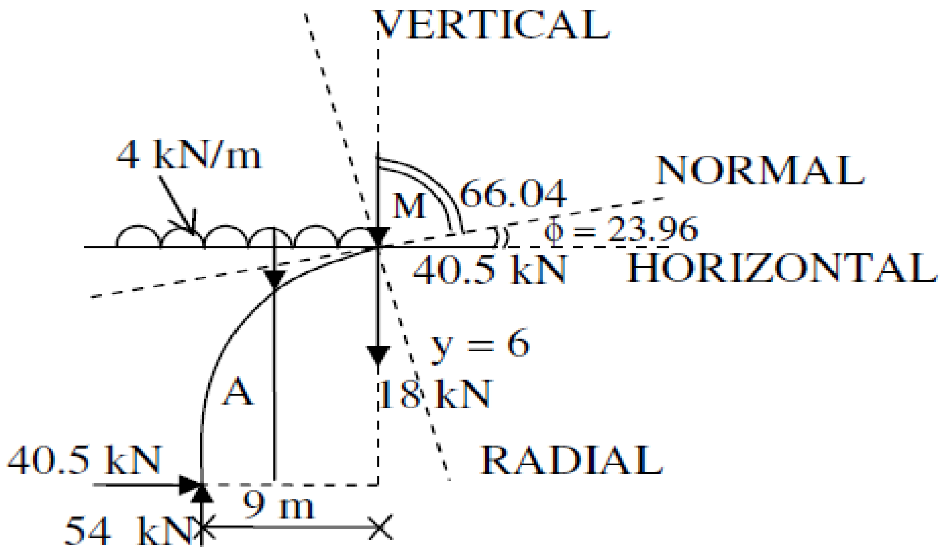
$$V_B = 18 \text{ kN} \quad \therefore V_A = 54 \text{ kN}$$

$$M_c = 0$$

$$+V_B \text{ x } 18 - H_B \text{ x } 8 = 0$$

$$H_B = 40.5 \text{ kN}$$

$$H_A = 40.5 \text{ kN}$$



BM at M =

- 40.5 x 6 + 54 x 9

- 4 x 9 x 4.5

= 81 kN.m

$$y = \frac{4hx}{L^2}(L - x)$$

$$y = \frac{4 \text{ x } 8 \text{ x } 9}{36^2}(36 - 9)$$

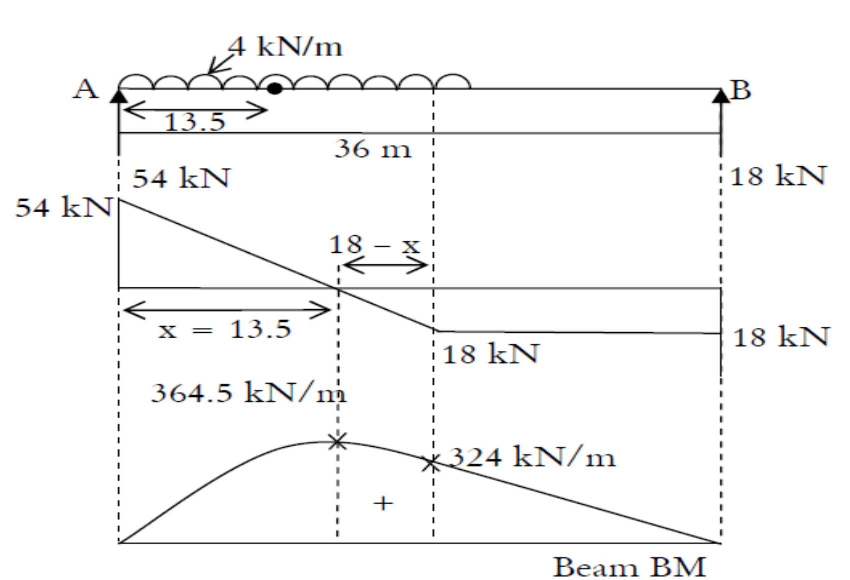
$$y = 6\text{m}$$

Shear force at M = + 54 – 4 x 9 = 18 kN (only vertical forces)

$$\tan\phi = \frac{4h}{L^2}(L-2x)$$

$$= \frac{4 \times 8}{36^2}(36 - 2 \times 9)$$

$$\phi = 23^0.96$$



Normal thrust = N = + 40.5 Cos 23.96 + 18 Cos (

= 44.32 kN

S = 40.5 Sin 23.96 – 18 Sin 66.04

S = - 0.0019 ≈ 0

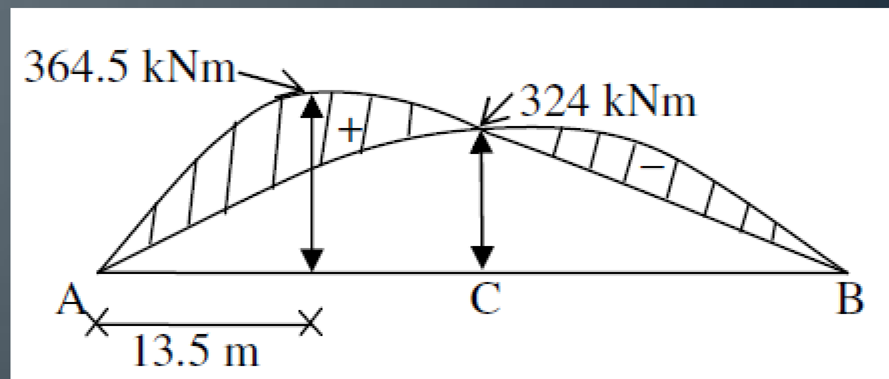
$$\frac{x}{54} = \frac{18-x}{18}$$

$$x = 54 - 3x$$

$$x = 13.5m$$

$$54 \times 13.5 - 4 \times 13.5 \times \frac{13.5}{2}$$

= 364.5 kNm



BMD